

CODON-ANTICODON INTERACTION and GENETIC CODE EVOLUTION

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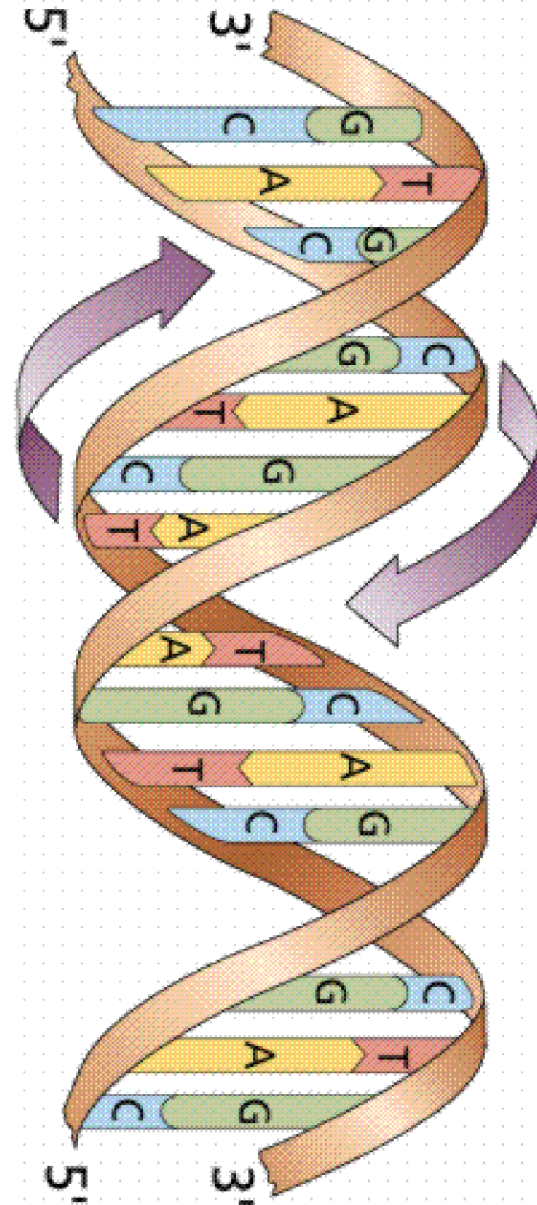
Plan:

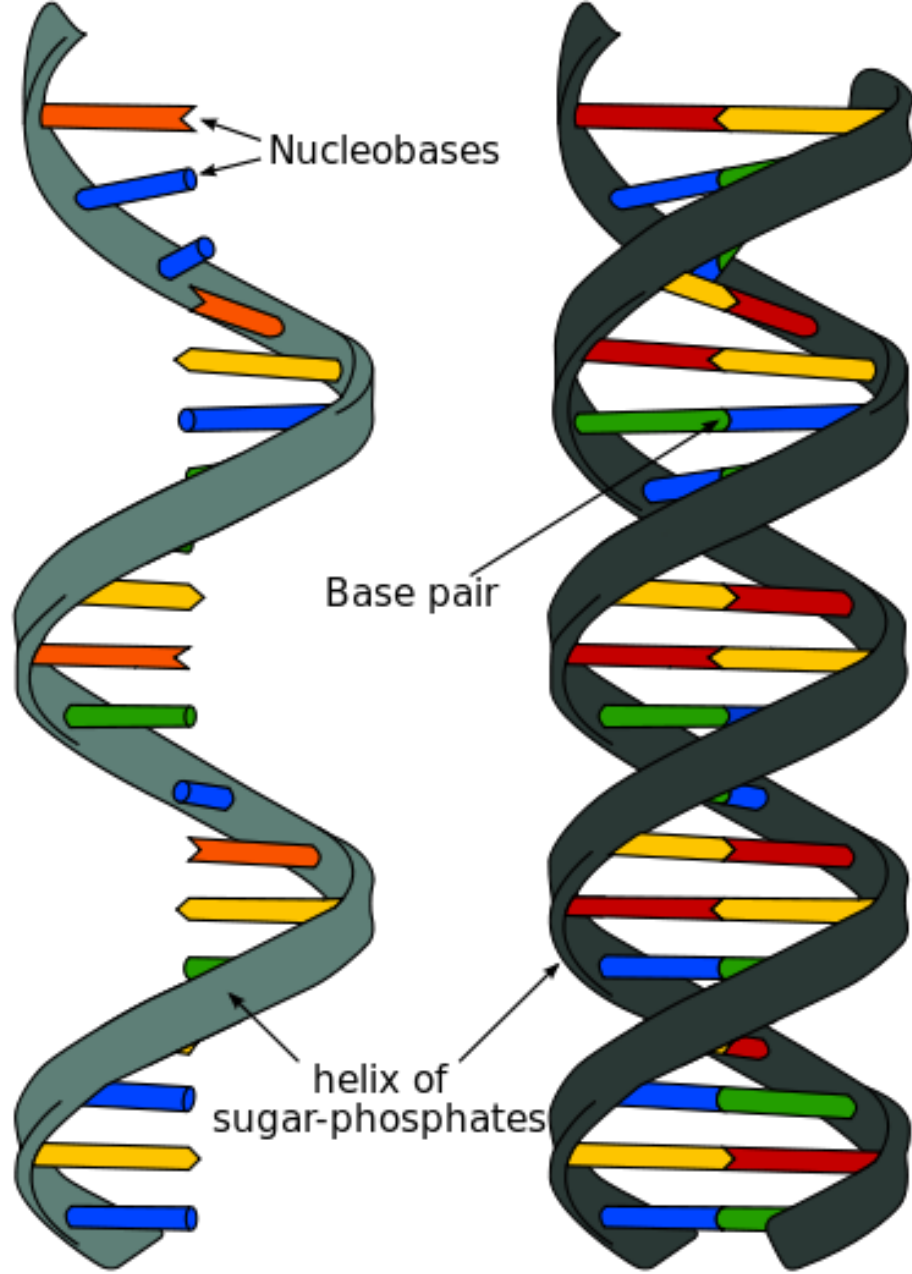
1. *Genetic code : a brief survey*
2. *Crystal Basis Model of the Genetic Code*
3. *Codon-anticodon Interaction*
4. *Evolution of the Genetic code*

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1- Genetic Code: *a brief survey*

The transmission of information from DNA (formed by 4 bases **C,G,A,T**) (**nucleotides**) to protein building is a complex process of transcription and translation.

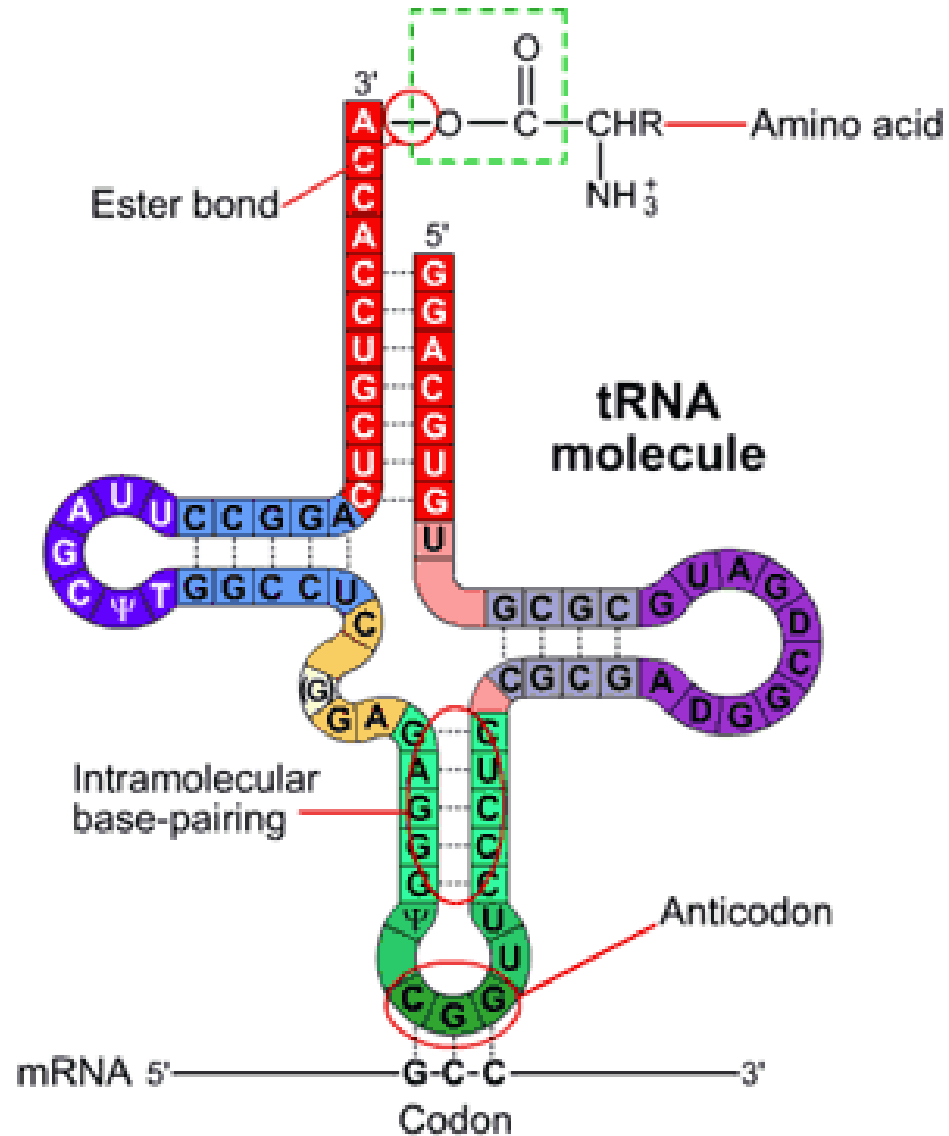




RNA
Ribonucleic acid

DNA
Deoxyribonucleic acid

From mRNA to tRNA and Amino acid



The Genetic Code

Triple of **4 nucleotides**
(**adenine (A)**, **guanine (G)**,
uracile (U), **cytosine (C)**)
→ **64 codons**

Number of **amino-acids** = 20

GENETIC CODE is degenerate

Standard code

61 codons code for **20 amino-acids** + **3 Stop**

The correspondence between **codons** and **amino-acids** is by multiplets
(of **synonymous codons**)

- n. 3 sextets
- n. 5 quartets
- n. 2 triplets
- n. 9 doublets
- n. 2 singlets

Standard Genetic Code

Second letter

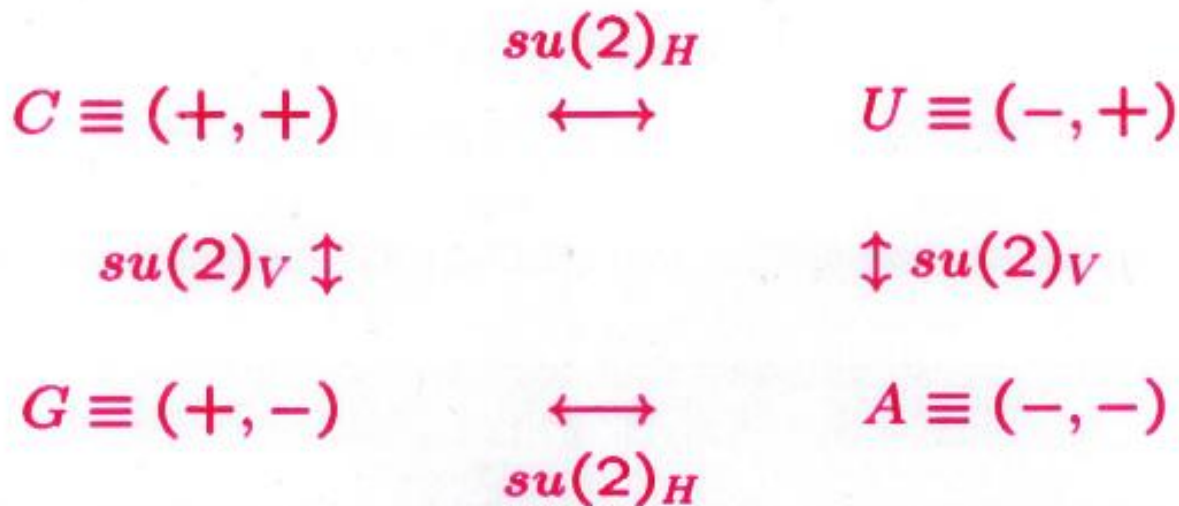
First letter

	U	C	A	G			
U	<div style="border: 1px solid black; padding: 2px;">UUU</div> <div style="border: 1px solid black; padding: 2px;">UUC</div> Phenylalanine	<div style="border: 1px solid black; padding: 2px;">UCU</div> <div style="border: 1px solid black; padding: 2px;">UCC</div> <div style="border: 1px solid black; padding: 2px;">UCA</div> <div style="border: 1px solid black; padding: 2px;">UCG</div> Serine	<div style="border: 1px solid black; padding: 2px;">UAU</div> <div style="border: 1px solid black; padding: 2px;">UAC</div> Tyrosine	<div style="border: 1px solid black; padding: 2px;">UGU</div> <div style="border: 1px solid black; padding: 2px;">UGC</div> Cysteine	U		
C	<div style="border: 1px solid black; padding: 2px;">UUA</div> <div style="border: 1px solid black; padding: 2px;">UUG</div> Leucine	<div style="border: 1px solid black; padding: 2px;">CUU</div> <div style="border: 1px solid black; padding: 2px;">CUC</div> <div style="border: 1px solid black; padding: 2px;">CUA</div> <div style="border: 1px solid black; padding: 2px;">CUG</div> Leucine	<div style="border: 1px solid black; padding: 2px;">CCU</div> <div style="border: 1px solid black; padding: 2px;">CCC</div> <div style="border: 1px solid black; padding: 2px;">CCA</div> <div style="border: 1px solid black; padding: 2px;">CCG</div> Proline	<div style="border: 1px solid black; padding: 2px;">CAA</div> <div style="border: 1px solid black; padding: 2px;">CAG</div> Glutamine	<div style="border: 1px solid black; padding: 2px;">UGA</div> Stop codon	<div style="border: 1px solid black; padding: 2px;">UGG</div> Tryptophan	A
A	<div style="border: 1px solid black; padding: 2px;">AUU</div> <div style="border: 1px solid black; padding: 2px;">AUC</div> <div style="border: 1px solid black; padding: 2px;">AUA</div> Isoleucine	<div style="border: 1px solid black; padding: 2px;">ACU</div> <div style="border: 1px solid black; padding: 2px;">ACC</div> <div style="border: 1px solid black; padding: 2px;">ACA</div> <div style="border: 1px solid black; padding: 2px;">ACG</div> Threonine	<div style="border: 1px solid black; padding: 2px;">AAU</div> <div style="border: 1px solid black; padding: 2px;">AAC</div> Asparagine	<div style="border: 1px solid black; padding: 2px;">AGU</div> <div style="border: 1px solid black; padding: 2px;">AGC</div> Serine	U		
G	<div style="border: 1px solid black; padding: 2px;">AUG</div> Methionine; initiation codon	<div style="border: 1px solid black; padding: 2px;">AAU</div> <div style="border: 1px solid black; padding: 2px;">AAC</div> Asparagine	<div style="border: 1px solid black; padding: 2px;">AAA</div> <div style="border: 1px solid black; padding: 2px;">AAG</div> Lysine	<div style="border: 1px solid black; padding: 2px;">AGA</div> <div style="border: 1px solid black; padding: 2px;">AGG</div> Arginine	C		
	<div style="border: 1px solid black; padding: 2px;">GUU</div> <div style="border: 1px solid black; padding: 2px;">GUC</div> <div style="border: 1px solid black; padding: 2px;">GUA</div> <div style="border: 1px solid black; padding: 2px;">GUG</div> Valine	<div style="border: 1px solid black; padding: 2px;">GCU</div> <div style="border: 1px solid black; padding: 2px;">GCC</div> <div style="border: 1px solid black; padding: 2px;">GCA</div> <div style="border: 1px solid black; padding: 2px;">GCG</div> Alanine	<div style="border: 1px solid black; padding: 2px;">GAU</div> <div style="border: 1px solid black; padding: 2px;">GAC</div> Aspartic acid	<div style="border: 1px solid black; padding: 2px;">GAA</div> <div style="border: 1px solid black; padding: 2px;">GAG</div> Glutamic acid	<div style="border: 1px solid black; padding: 2px;">GGU</div> <div style="border: 1px solid black; padding: 2px;">GGC</div> <div style="border: 1px solid black; padding: 2px;">GGA</div> <div style="border: 1px solid black; padding: 2px;">GGG</div> Glycine	A	
					G		

2- Crystal Basis Model of G.C.

4 bases : purines : (A, G) and bases
 pyrimidines : (C, T/U) complementarity

$\Rightarrow (\frac{1}{2}, \frac{1}{2})$ representation of $SU(2) \times SU(2)$.



Analogy between quark (q) and baryon ($3q$)
and base (b) and codon ($3b$).

But :

$$|p\rangle \sim |uud\rangle + |udu\rangle + |duu\rangle \text{ (implicit spin structure)}$$

while:

$UAG \neq AUG$ in codons. *no mixing*

\implies Limit of the quantum (deformed) algebra
 $\mathcal{U}_q[sl(2) \oplus sl(2)]$ when $q \rightarrow 0$

(remind: $q \rightarrow 1$ usual $\mathcal{U}[sl(2) \oplus sl(2)]$).

Then:

tensorial product of representations

=

“pure” states of constituent states
in **crystal bases**.

$U_q(sl(2))$

$$[J_z, J_{\pm}] = \pm J_{\pm}$$

$$[J_+, J_-] = \frac{q^{J_z} - q^{-J_z}}{q^{1/2} - q^{-1/2}}$$

$$q = 1 \Rightarrow sl(2)$$

$$q = 0 \Rightarrow \text{Crystal Basis}$$

In our model: a codon $\in (\frac{1}{2}, \frac{1}{2}) \otimes (\frac{1}{2}, \frac{1}{2}) \otimes (\frac{1}{2}, \frac{1}{2})$.

product of two representations

$$(\frac{1}{2}, \frac{1}{2}) \otimes (\frac{1}{2}, \frac{1}{2}) = (1, 1) \oplus (1, 0) \oplus (0, 1) \oplus (0, 0)$$

$$\begin{array}{l} \rightarrow su(2)_H \quad (J_H, J_V) = (0, 0) \quad (CA) \quad (1, 0) \quad (CG \quad UG \quad UA) \\ \downarrow \\ su(2)_V \quad (0, 1) \quad \begin{pmatrix} CU \\ GU \\ GA \end{pmatrix} \quad (1, 1) \quad \begin{pmatrix} CC & UC & UU \\ GC & AC & AU \\ GG & AG & AA \end{pmatrix} \end{array}$$

Property:

quadruplets $\mathcal{Q} \rightarrow 0$

(as well as those in sextets)

s.t.

$$J_{3,H} > 0$$

or

$$J_{3,H} = 0 \text{ and } J_{3,V} \geq 0, J_V \neq 0$$

doublets $\mathcal{Q} \leftarrow 0$

(and others: triplet, singlets)

s.t.

$$J_{3,H} < 0$$

or

$$J_{3,H} = 0 \text{ and } J_{3,V} < 0 \text{ or } J_V = 0$$

$$\mathcal{Q} = J_{3,H} + \frac{1}{4} C_V (J_{3,V} + 2) - \frac{1}{4}$$

product of three representations = codons

$$\left(\frac{1}{2}, \frac{1}{2}\right) \otimes \left(\frac{1}{2}, \frac{1}{2}\right) \otimes \left(\frac{1}{2}, \frac{1}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right) \oplus 2 \left(\frac{3}{2}, \frac{1}{2}\right) \oplus 2 \left(\frac{1}{2}, \frac{3}{2}\right) \oplus 4 \left(\frac{1}{2}, \frac{1}{2}\right)$$

$$\left(\frac{3}{2}, \frac{3}{2}\right) \equiv \begin{pmatrix} CCC & UCC & UUC & UUU \\ GCC & ACC & AUC & AUU \\ GGC & AGC & AAC & AAU \\ GGG & AGG & AAG & AAA \end{pmatrix}$$

$$\left(\frac{3}{2}, \frac{1}{2}\right) \equiv \begin{pmatrix} CCG & UCG & UUG & UUA \\ GCG & ACG & AUG & AUA \end{pmatrix}$$

$$\left(\frac{3}{2}, \frac{1}{2}\right)' \equiv \begin{pmatrix} CGC & UGC & UAC & UAU \\ CGG & UGG & UAG & UAA \end{pmatrix}$$

$$\left(\frac{1}{2}, \frac{3}{2}\right) \equiv \begin{pmatrix} CCU & UCU \\ GCU & ACU \\ GGU & AGU \\ GGA & AGA \end{pmatrix}$$

$$\left(\frac{1}{2}, \frac{3}{2}\right)' \equiv \begin{pmatrix} CUC & CUU \\ GUC & GUU \\ GAC & GAU \\ GAG & GAA \end{pmatrix}$$

$$\left(\frac{1}{2}, \frac{1}{2}\right) \equiv \begin{pmatrix} CCA & UCA \\ GCA & ACA \end{pmatrix}$$

$$\left(\frac{1}{2}, \frac{1}{2}\right)' \equiv \begin{pmatrix} CGU & UGU \\ CGA & UGA \end{pmatrix}$$

$$\left(\frac{1}{2}, \frac{1}{2}\right)'' \equiv \begin{pmatrix} CUG & CUA \\ GUG & GUA \end{pmatrix}$$

$$\left(\frac{1}{2}, \frac{1}{2}\right)''' \equiv \begin{pmatrix} CAC & CAU \\ CAG & CAA \end{pmatrix}$$

Cod.	a.a.	J _H	J _V	Cod.	a.a.	J _H	J _V
CCC	Pro	3/2	3/2	UCC	Ser	3/2	3/2
CCU	Pro	(1/2	3/2) ¹	UCU	Ser	(1/2	3/2) ¹
CCG	Pro	(3/2	1/2) ¹	UCG	Ser	(3/2	1/2) ¹
CCA	Pro	(1/2	1/2) ¹	UCA	Ser	(1/2	1/2) ¹
CUC	Leu	(1/2	3/2) ²	UUC	Phe	3/2	3/2
CUU	Leu	(1/2	3/2) ²	UUU	Phe	3/2	3/2
CUG	Leu	(1/2	1/2) ³	UUG	Leu	(3/2	1/2) ¹
CUA	Leu	(1/2	1/2) ³	UUA	Leu	(3/2	1/2) ¹
CGC	Arg	(3/2	1/2) ²	UGC	Cys	(3/2	1/2) ²
CGU	Arg	(1/2	1/2) ²	UGU	Cys	(1/2	1/2) ²
CGG	Arg	(3/2	1/2) ²	UGG	Trp	(3/2	1/2) ²
CGA	Arg	(1/2	1/2) ²	UGA	Trp	(1/2	1/2) ²
CAC	His	(1/2	1/2) ⁴	UAC	Tyr	(3/2	1/2) ²
CAU	His	(1/2	1/2) ⁴	UAU	Tyr	(3/2	1/2) ²
CAG	Gln	(1/2	1/2) ⁴	UAG	Ter	(3/2	1/2) ²
CAA	Gln	(1/2	1/2) ⁴	UAA	Ter	(3/2	1/2) ²
GCC	Ala	3/2	3/2	ACC	Thr	3/2	3/2
GCU	Ala	(1/2	3/2) ¹	ACU	Thr	(1/2	3/2) ¹
GUG	Ala	(3/2	1/2) ¹	ACG	Thr	(3/2	1/2) ¹
GCA	Ala	(1/2	1/2) ¹	ACA	Thr	(1/2	1/2) ¹
GUC	Val	(1/2	3/2) ²	AUC	Ile	3/2	3/2
GUU	Val	(1/2	3/2) ²	AUU	Ile	3/2	3/2
GUG	Val	(1/2	1/2) ³	AUG	Met	(3/2	1/2) ¹
GUA	Val	(1/2	1/2) ³	AUA	Ile	(3/2	1/2) ¹
GGC	Gly	3/2	3/2	AGC	Ser	3/2	3/2
GGU	Gly	(1/2	3/2) ¹	AGU	Ser	(1/2	3/2) ¹
GGG	Gly	3/2	3/2	AGG	Arg	3/2	3/2
GGA	Gly	(1/2	3/2) ¹	AGA	Arg	(1/2	3/2) ¹
GAC	Asp	(1/2	3/2) ²	AAC	Asn	3/2	3/2
GAU	Asp	(1/2	3/2) ²	AAU	Asn	3/2	3/2
GAG	Glu	(1/2	3/2) ²	AAG	Lys	3/2	3/2
GAA	Glu	(1/2	3/2) ²	AAA	Lys	3/2	3/2

- ***Applications** of this model provided in a series of papers (L.Frappat, A.Sciarrino and P.S.) in the years 1998-2005. Among them:*
- *Study of **codon usage probabilities**, elaboration of sum rules.*
- *Relations between **physico-chemical properties of amino-acids** (a.a.) and predictions.*
- *More mathematical aspects: operator relating a.a. and codons for any known genetic code; attempts to describe mutations, etc.*

3- Codon-anticodon Interaction

Position of the problem:

codon: XYZ ---- anti-codon: Z'Y'X'

with nucleotids Z',Y',X' associated to Z, Y, X

- In tRNA process, codon –anticodon pairing **does not** follow the usual Watson-Crick pattern (i.e. pairing C --- G , U --- A).
- This leads Crick (1966) to propose the **wobble hypothesis** :

A specified anti-codon can recognize more than one codon differing only in the third nucleotide.

i.e. standard pairing for X--X' and Y—Y' while Z' may pair to different Z.

- Two main hypotheses proposed in this context:
 - 1- for doublets, the first nucleotide Z' in anticodon should have G (resp. U) to read for codon with Y (resp. R) in third position Z.
 - 2- the chosen anticodon is the one with first position nucleotide pairing the (third position of the) most abundant codon among synonymous codons.

(Y= C, U pyrimidine, R= G,A purine)

Considering the Mitochondrial Code:

there are: 2 sextets,
6 quadruplets
12 doublets of codons specifying the 20 amino-acids.

So, a minimum number of 22 anticodons is needed.

And this appears to be the case in mitochondria of animals
(*Sprinzi et al., 1998*)

Data seem to confirm the empirical rule just above.

It is this set of data that we will consider now in the framework of
the Crystal Basis Model.

The Minimum Principle

Consider the operator:

$$T(\text{anticodon, codon}) = c_H \vec{J}_H^c \cdot \vec{J}_H^a + c_V \vec{J}_V^c \cdot \vec{J}_V^a$$

where :

$$\vec{J}^c \cdot \vec{J}^a = \frac{1}{2} \{ (\vec{J}^c + \vec{J}^a)^2 - (\vec{J}^c)^2 - (\vec{J}^a)^2 \}$$

and:

$$\vec{J} = (J_1, J_2, J_3) \text{ generators of } \text{su}(2)_{\substack{H \\ V}} \text{ group.}$$

Then define:

- for quadruplets: taking as an example Val (GUN; N=C,U,G,A) and as a possible anticodon CAC:

$$T_{\text{aver.}}(\text{CAC, Val}) = \sum_N P_N^{\text{val}} T(\text{CAC, GUN})$$

with : $P_C^q + P_U^q + P_G^q + P_A^q = 1$

- for doublets : taking as an ex. Asp (GAC, GAU) and as a possible anticodon CUC:

$$T_{\text{aver.}} (\text{CUC, Asp}) = \sum_{\mathbf{y}} P_{\mathbf{y}}^{\text{Asp}} \cdot T (\text{CUC, GUY})$$

$$\text{with : } P_{\mathbf{y}}^d = P_c^d + P_u^d = 1 \quad (\text{and } P_{\mathbf{R}}^d = P_G^d + P_A^d = 1)$$

Question:

Can we determine $c_{\mathbf{H}}$ and $c_{\mathbf{V}}$ such that for each given quadruplet (or doublet) of codons, the anticodon minimizing $T_{\text{aver.}}$ is the one given by the data ?

Let us remind that the possible anticodons to the codon XY N is N'Y'X' with X -- X' and Y --Y' related by the "usual pairing" (i.e. C -- G, U -- A) and N' is any nucleotide C,G,U, A).

In Vertebral Mitochondrial Code:

most used anticodons for mitochondria of animals
(*Sprinzi et al, 1998*)

codon	a.a.	J_H	J_V	$J_{3,H}$	$J_{3,V}$	anticodon	codon	a.a.	J_H	J_V	$J_{3,H}$	$J_{3,V}$	anticodon
CCC	P	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	UGG	UCC	S	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	UGA
CCU	P	$(\frac{1}{2} \frac{3}{2})^1$		$\frac{1}{2}$	$\frac{3}{2}$		UCU	S	$(\frac{1}{2} \frac{3}{2})^1$		$-\frac{1}{2}$	$\frac{3}{2}$	
CCG	P	$(\frac{3}{2} \frac{1}{2})^1$		$\frac{3}{2}$	$\frac{1}{2}$		UCG	S	$(\frac{3}{2} \frac{1}{2})^1$		$\frac{1}{2}$	$\frac{1}{2}$	
CCA	P	$(\frac{1}{2} \frac{1}{2})^1$		$\frac{1}{2}$	$\frac{1}{2}$		UCA	S	$(\frac{1}{2} \frac{1}{2})^1$		$-\frac{1}{2}$	$\frac{1}{2}$	
CUC	L	$(\frac{1}{2} \frac{3}{2})^2$		$\frac{1}{2}$	$\frac{3}{2}$	UAG	UUC	F	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	GAA
CUU	L	$(\frac{1}{2} \frac{3}{2})^2$		$-\frac{1}{2}$	$\frac{3}{2}$		UUU	F	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	$\frac{3}{2}$	
CUG	L	$(\frac{1}{2} \frac{1}{2})^3$		$\frac{1}{2}$	$\frac{1}{2}$		UUG	L	$(\frac{3}{2} \frac{1}{2})^1$		$-\frac{1}{2}$	$\frac{1}{2}$	
CUA	L	$(\frac{1}{2} \frac{1}{2})^3$		$-\frac{1}{2}$	$\frac{1}{2}$		UUA	L	$(\frac{3}{2} \frac{1}{2})^1$		$-\frac{3}{2}$	$\frac{1}{2}$	
CGC	R	$(\frac{3}{2} \frac{1}{2})^2$		$\frac{3}{2}$	$\frac{1}{2}$	UCG	UGC	C	$(\frac{3}{2} \frac{1}{2})^2$		$\frac{1}{2}$	$\frac{1}{2}$	GCA
CGU	R	$(\frac{1}{2} \frac{1}{2})^2$		$\frac{1}{2}$	$\frac{1}{2}$		UGU	C	$(\frac{1}{2} \frac{1}{2})^2$		$-\frac{1}{2}$	$\frac{1}{2}$	
CGG	R	$(\frac{3}{2} \frac{1}{2})^2$		$\frac{3}{2}$	$-\frac{1}{2}$		UGG	W	$(\frac{3}{2} \frac{1}{2})^2$		$\frac{1}{2}$	$-\frac{1}{2}$	
CGA	R	$(\frac{1}{2} \frac{1}{2})^2$		$\frac{1}{2}$	$-\frac{1}{2}$		UGA	W	$(\frac{1}{2} \frac{1}{2})^2$		$-\frac{1}{2}$	$-\frac{1}{2}$	
CAC	H	$(\frac{1}{2} \frac{1}{2})^4$		$\frac{1}{2}$	$\frac{1}{2}$	GUG	UAC	Y	$(\frac{3}{2} \frac{1}{2})^2$		$-\frac{1}{2}$	$\frac{1}{2}$	GUA
CAU	H	$(\frac{1}{2} \frac{1}{2})^4$		$-\frac{1}{2}$	$\frac{1}{2}$		UAU	Y	$(\frac{3}{2} \frac{1}{2})^2$		$-\frac{3}{2}$	$\frac{1}{2}$	
CAG	Q	$(\frac{1}{2} \frac{1}{2})^4$		$\frac{1}{2}$	$-\frac{1}{2}$	UUG	UAG	Ter	$(\frac{3}{2} \frac{1}{2})^2$		$-\frac{1}{2}$	$-\frac{1}{2}$	—
CAA	Q	$(\frac{1}{2} \frac{1}{2})^4$		$-\frac{1}{2}$	$-\frac{1}{2}$		UAA	Ter	$(\frac{3}{2} \frac{1}{2})^2$		$-\frac{3}{2}$	$-\frac{1}{2}$	
GCC	A	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	UGC	ACC	T	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	UGU
GCU	A	$(\frac{1}{2} \frac{3}{2})^1$		$\frac{1}{2}$	$\frac{1}{2}$		ACU	T	$(\frac{1}{2} \frac{3}{2})^1$		$-\frac{1}{2}$	$\frac{1}{2}$	
GCG	A	$(\frac{3}{2} \frac{1}{2})^1$		$\frac{3}{2}$	$-\frac{1}{2}$		ACG	T	$(\frac{3}{2} \frac{1}{2})^1$		$\frac{1}{2}$	$-\frac{1}{2}$	
GCA	A	$(\frac{1}{2} \frac{1}{2})^1$		$\frac{1}{2}$	$-\frac{1}{2}$		ACA	T	$(\frac{1}{2} \frac{1}{2})^1$		$-\frac{1}{2}$	$-\frac{1}{2}$	
GUC	V	$(\frac{1}{2} \frac{3}{2})^2$		$\frac{1}{2}$	$\frac{1}{2}$	UAC	AUC	I	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	GAU
GUU	V	$(\frac{1}{2} \frac{3}{2})^2$		$-\frac{1}{2}$	$\frac{1}{2}$		AUU	I	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	
GUG	V	$(\frac{1}{2} \frac{1}{2})^3$		$\frac{1}{2}$	$-\frac{1}{2}$		AUG	M	$(\frac{3}{2} \frac{1}{2})^1$		$-\frac{1}{2}$	$-\frac{1}{2}$	
GUA	V	$(\frac{1}{2} \frac{1}{2})^3$		$-\frac{1}{2}$	$-\frac{1}{2}$		AUA	M	$(\frac{3}{2} \frac{1}{2})^1$		$-\frac{3}{2}$	$-\frac{1}{2}$	
GGC	G	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	UCC	AGC	S	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	GCU
GGU	G	$(\frac{1}{2} \frac{3}{2})^1$		$\frac{1}{2}$	$-\frac{1}{2}$		AGU	S	$(\frac{1}{2} \frac{3}{2})^1$		$-\frac{1}{2}$	$-\frac{1}{2}$	
GGG	G	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$		AGG	Ter	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	
GGA	G	$(\frac{1}{2} \frac{3}{2})^1$		$\frac{1}{2}$	$-\frac{3}{2}$		AGA	Ter	$(\frac{1}{2} \frac{3}{2})^1$		$-\frac{1}{2}$	$-\frac{3}{2}$	
GAC	D	$(\frac{1}{2} \frac{3}{2})^2$		$\frac{1}{2}$	$-\frac{1}{2}$	GUC	AAC	N	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	GUU
GAU	D	$(\frac{1}{2} \frac{3}{2})^2$		$-\frac{1}{2}$	$-\frac{1}{2}$		AAU	N	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{1}{2}$	
GAG	E	$(\frac{1}{2} \frac{3}{2})^2$		$\frac{1}{2}$	$-\frac{3}{2}$	UUC	AAG	K	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	UUU
GAA	E	$(\frac{1}{2} \frac{3}{2})^2$		$-\frac{1}{2}$	$-\frac{3}{2}$		AAA	K	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	

Results:

- for quadruplets: choose simply $c_H > 0$ and $c_V < 0$ to be in accordance with data.

-for doublets: choose $c_V > 0$
and the sign of c_H such that:

c_H is $>$ for the doublets : UUY, UAY, AU Y , AA Y
CAR, UGR, AGR, GAR

c_H is $<$ for the other doublets: UUR, UAR, AU R , AA R
CAY, UGY, AG Y , GA Y

(c_H of opposite sign for two doublets with same dinucleotide but ending with a purine or a pyrimidine).

For doublets, we remark that, with the choice of sign of c_H above specified and $c_V > 0$ for all a.a., the anticodons minimizing the average value of \mathcal{T} are in agreement with the observed anticodon, see (Sprinzl et al. , 1998) and Table 2. We summarize in Table 1 the results for the doublets.

a.a	sign c_H	anticodon	note
His	-	GUG	$P_C^d > 0,25$
Gln	+	UUG	$P_C^d > 0,25$
Phe	-	GAA	
Leu	+	UAA	
Cys	+	GCA	
Trp	-	UCA	
Tyr	-	GUA	
Ser	+	GCU	
Asp	+	GUC	$P_C^d > 0,25$
Glu	-	UUC	$P_C^d > 0,25$
Ile	+	GAU	
Met	-	CAU	
Asn	-	GUU	
Lys	+	UUU	

Conclusion

- Anticodons minimizing the conjectured operator T_{aver} in **very good agreement with the observed ones** for mitochondria of animals.
- Results depending only of the **sign of two coupling constants**.
- One may expect **a more complicated pattern in the general case**, following the biological species. Might happen that the « universal » feature of C_V and C_H should be released and T expressions modified.
- Then the crystal basis offers a lot of possibilities, for ex. adjunction of a term of « **spin-spin** » interaction of the type:

$$g_H J_{H,3}^e \cdot J_{H,3}^a + g_V J_{V,3}^e \cdot J_{V,3}^a$$

4- Evolution of the Genetic Code

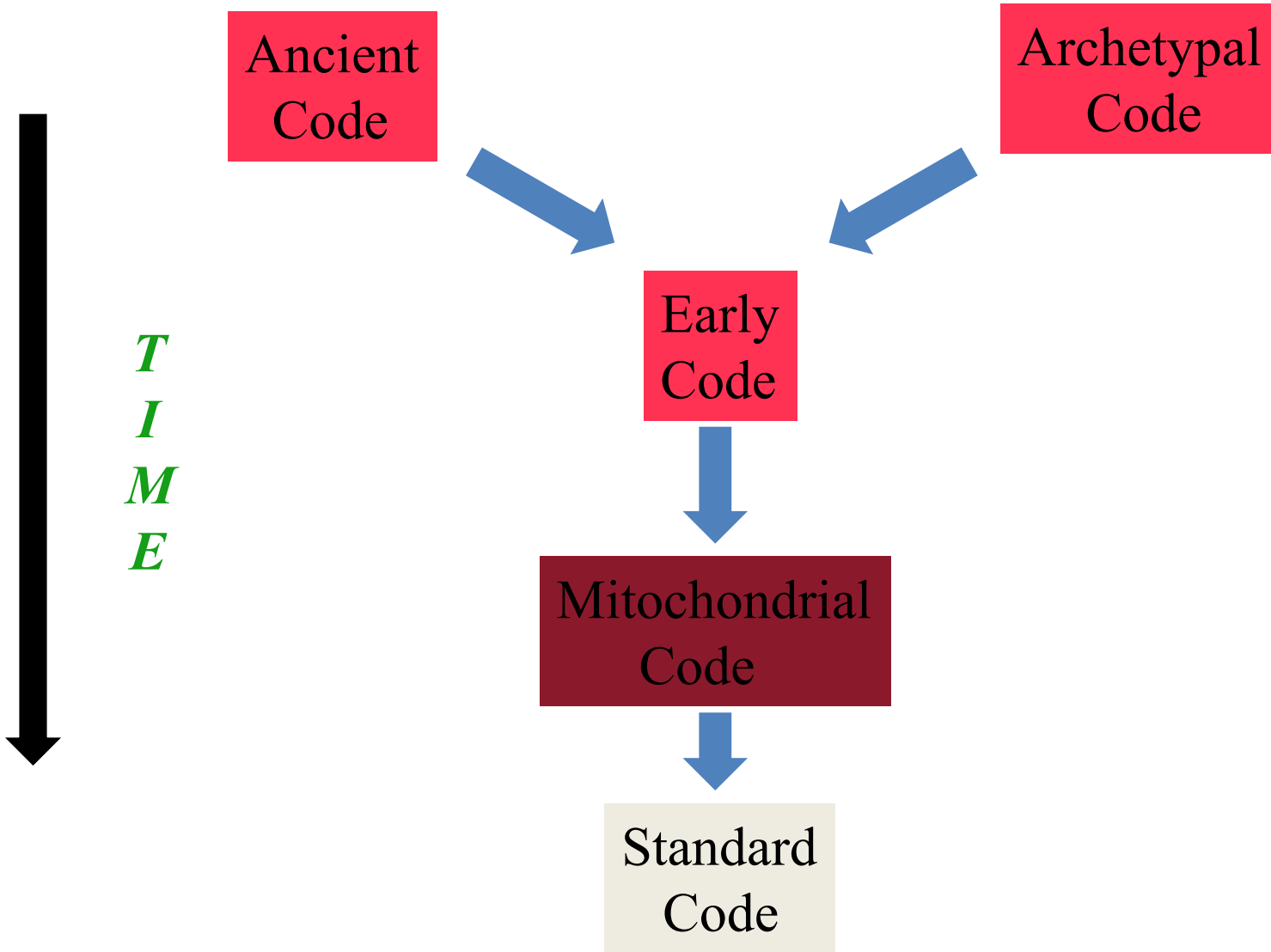
The genetic code has undergone an **evolutionary process**

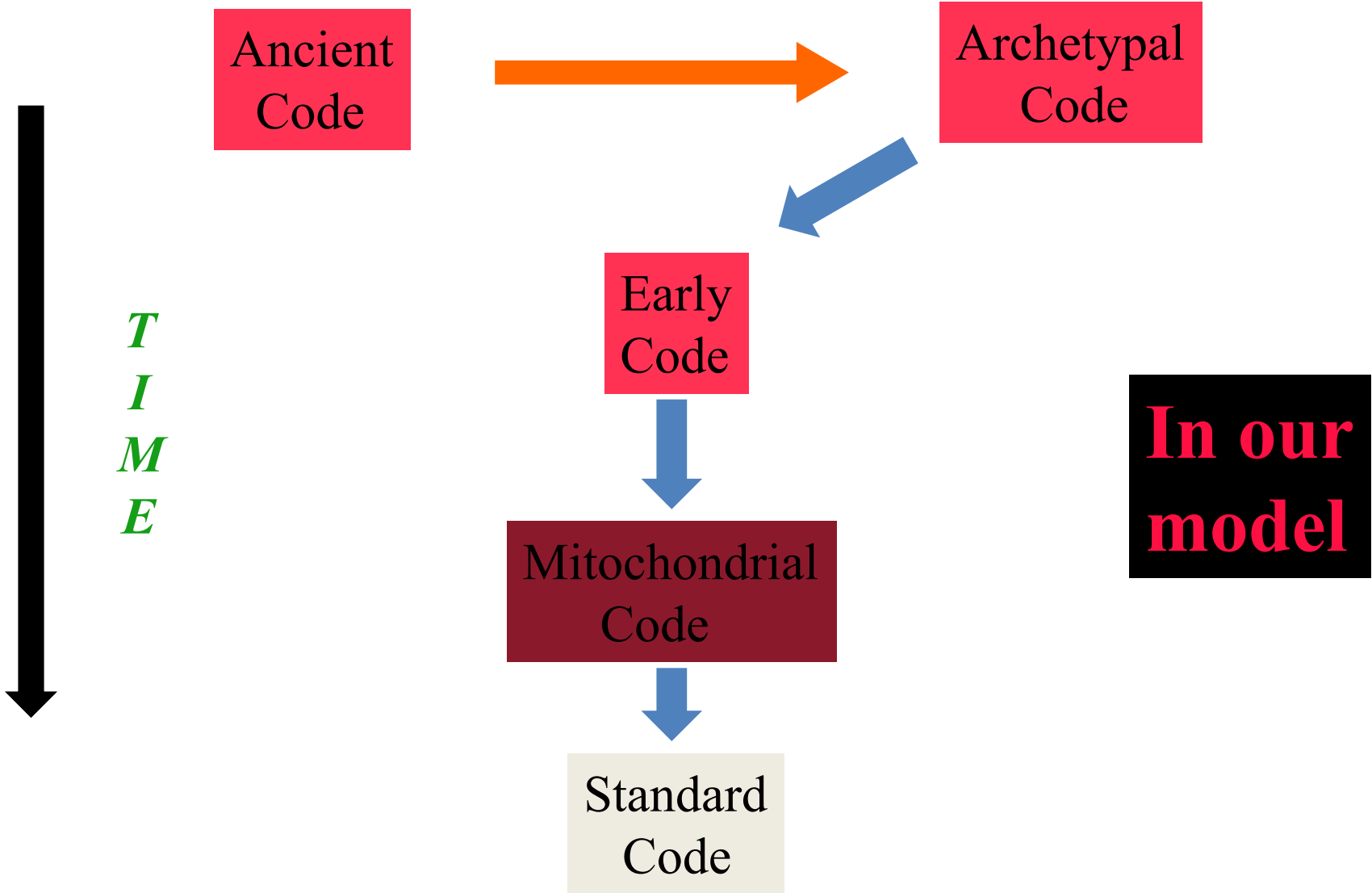
- An evolutionary theory is the « codon capture theory » (review Ohama et al. (2008)):

. *number of the encoded amino acids is kept **constant**, equal to **20**.*

. *the **coding codons change**, key role in this process being played by the **anticodon**.*

In this context, 3 main codes: the Ancient and Early Codes, and an alternative: the Archetypal Code.





A comparison and a proposition

Differences:

- In **Ancient** code, 20 codons and 20 anti-codons.
- In **Early** code, all codons involved and 20 anti-codons.
- In **Archetypal** code, all codons but 16 anticodons .

A proposal: **Minimisation Conjecture** in following way:

- **T** (anticodon-codon) in **Ancient** Code :
with Watson-Crick pairing between nucleotids (i.e. C-G , U-A)

- **T aver**(anticodon-codon) in **Archetypal** and **Early** Codes
with « wobble » mechanism.

Ancient code

in bold the differences with Ohama(2008)

a.a	codon	anticodon
Pro	CCG	CGG
Leu	CUA	UAG
Arg	CGG	CCG
Ala	GCG	CGC
Val	GUA	UAC
Gly	GGG	CCC
Ser	UCG	CGA
Thr	ACG	CGU
His	CAC	GUG
Gln	CAG	CUG
Phe	UUC	GAA
Cys	UGU	ACA
Trp	UGA	UCA
Tyr	UAC	GUA
Asp	GAC	GUC
Glu	GAG	CUC
Ile	AUC	GAU
Met	AUG	CAU
Asn	AAC	GUU
Lys	AAG	CUU

Table 2: The couple of codon-anticodon which minimizes the operator \mathcal{T} with $c_V < 0$ and $c_H < 0$ for the strong dinucleotides (first 8 rows) and c_V underdetermined $c_H > 0$ for the weak dinucleotides in the Ancient Genetic Code.

Archetypal code

a.a	codon	anticodon	sign c_H	sign c_V	note
Pro	CCN	UGG	+	-	
Leu	CUN	UAG	+	-	
Arg	CGN	UCG	+	-	
Ala	GCN	UGC	+	-	
Val	GUN	UAC	+	-	
Gly	GGN	UCC	+	-	
Ser	UCN	UGA	+	-	
Thr	ACN	UGU	+	-	
His/Gln	CAN	UUG	+	-	$P_S > 1/4$
Phe/Leu'	UUN	UAA	-	-	
Cys/Trp	UGN	UCA	+	+	
Tyr	UAY	GUA	+	+	
Asp/Glu	GAN	UUC	+	-	$P_S > 1/4$
Ile/Met	AUN	UAU	-	-	$P_Y > 1/8$
Asn/Lys	AAN	UUU	-	-	
Ser'/Arg'	AGN	UCU	+	-	

Table 3: Sign of coupling constants minimizing the operator \mathcal{T} , averaged over the codons, for any amino acid encoded in the Archetypal Genetic Code. We denote by a prime the a.a. encoded by the sub-part of the sextet corresponding to a doublet.

Early Code

a.a	codon	anticodon	sign c_H	sign c_V	note
Pro	CCN	UGG	+	-	
Leu	CUN	UAG	+	-	$P_S > 1/4$
Arg	CGN	UCG	+	-	
Ala	GCN	UGC	+	-	
Val	GUN	UAC	+	-	$P_S > 1/4$
Gly	GGN	UCC	+	-	
Ser	UCN	UGA	+	-	
Thr	ACN	UGU	+	-	$P_Y > 1/8$
His	CAY	GUG	+	+	$P_C > 3/8$
Gln	CAR	UUG	-	und.	$P_G < 1/4$
Phe	UUY	GAA	+	+	
Leu'	UUR	UAA	-	und.	
Cys	UGY	GCA	-	+	
Trp	UGR	UCA	+	und.	
Tyr	UAY	GUA	+	+	
Asp	GAY	GUC	+	+	$P_C > 1/4$
Glu	GAR	UUC	-	und.	$P_G < 1/4$
Ile	AUY	GAU	+	+	
Met	AUR	UAU	-	und.	
Asn	AAY	GUU	+	+	
Lys	AAR	UUU	-	und.	
Ser'	AGY	GCU	-	+	
Arg'	AGR	UCU	+	und.	

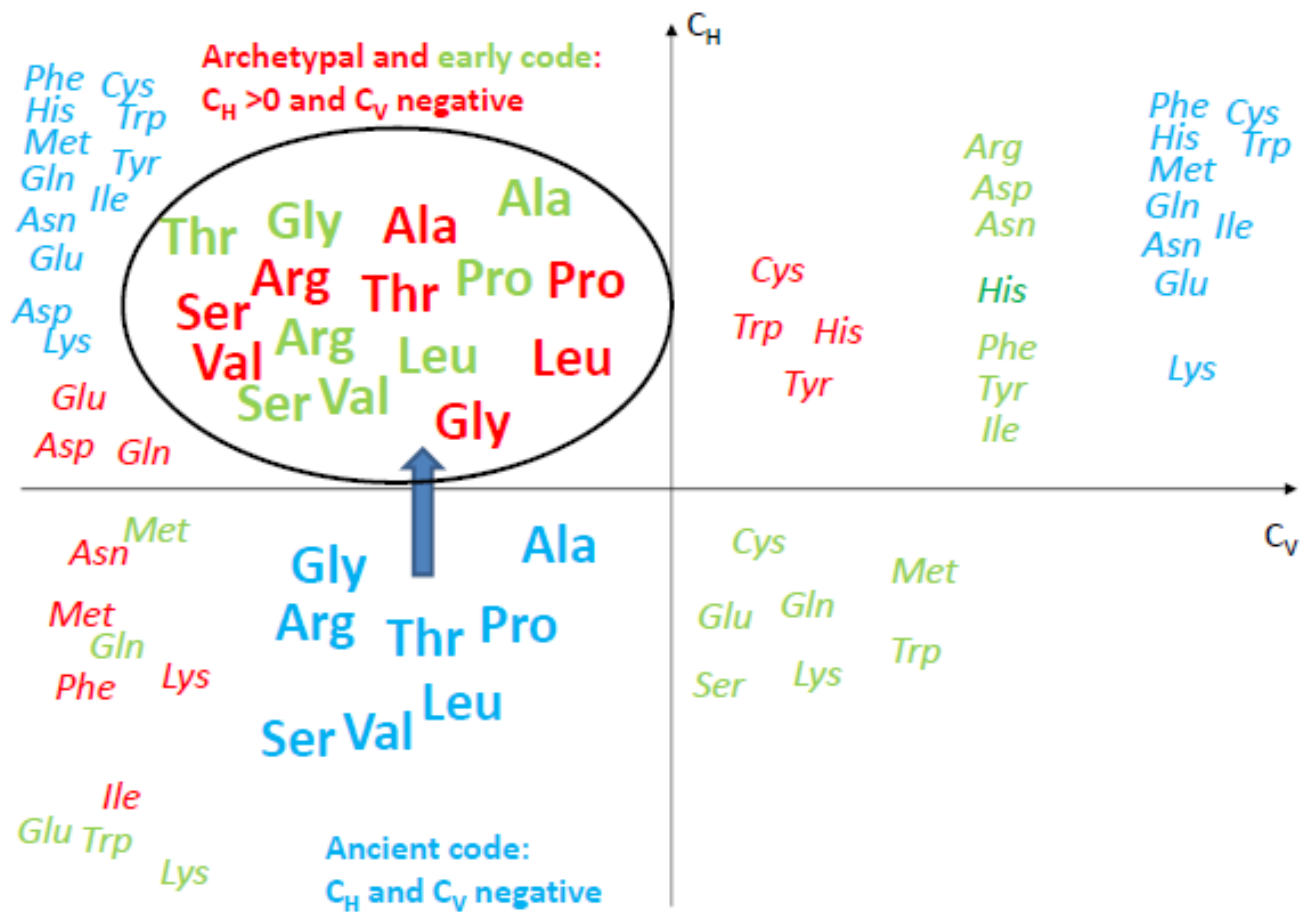
Table 4: Sign of coupling constants minimizing the operator \mathcal{T} , averaged over the codons, for any amino acid encoded in the Early Genetic Code.

Remark:

« evolution » of the signs of the Constants:

example of Asn/Lys

$$c_H^{AAN} > 0 \implies c_H^{AAN} < 0 \implies \begin{cases} c_H^{AAY} > 0 \\ c_H^{AAR} < 0 \end{cases}$$



Conclusion

- The « constants » C_V and C_H deserve a more precise study. Their *branching points* would correspond to the advent of different Genetic Codes, with the standard G.C. emerging as the one exhibiting *selective advantages*.
- Could we verify that the existing G.C., that is the branching point which has survived, satisfies the required *optimality conditions*?

Summary

- The *evolution* of the Genetic Code is analysed from the view-point of the *codon-anticodon interaction*.
- Imposing a *Minimum principle for the Interaction*, we determine, in the framework of the *Crystal Basis Model*, the structure of anticodons in the *Ancient, Archetypal and Early Genetic Codes*, that are all reconciled in a unique frame.
- The above obtained results, joined to previous ones, encourage us to introduce the notion of « *BIO-SPIN* » related to the $Uq(SU(2)+SU(2))$ group of our model.